Appendix 1: Rules for Exponents

(according to kwn)

1. $a^n a^m = a^{n+m}$

2. $\frac{a^n}{a^m} = a^{n-m}$

3. $(a^n)^m = a^{nm}$

4. $(ab)^n = a^n b^n$

4'. $(a^k b^p)^n = a^{kn} b^{pn}$

4". $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$

5. $\frac{a}{b}^{\,n} = \frac{a^n}{b^n}$

5'. $\frac{a^k}{b^p}^{\,n} = \frac{a^{kn}}{b^{pn}}$

5". $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

5.''' $\frac{a^k}{b^p}^{\,-n} = \frac{b^p}{a^k}^{\,n} = \frac{b^{pn}}{a^{kn}}$

6. $a^0 = 1$

7. $a^{-n} = \frac{1}{a^n}$

8. $\sqrt[n]{a} = a^{\frac{1}{n}}$

8'. $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$

9. $\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}$

10. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

The Number e.

$e = \underset{n\ 6\ 4}{\text{limit}}\,(1 + \frac{1}{n})^n$ • 2.718281828... It is a nonterminating, nonrepeating decimal, hence irrational.

Math 112 Discussion 1: Introduction & P.2

Intro: Do the numbers trick, give them the chocolate one to figure out.

Note 1 : You are expected to be able to do all the assigned problems from textbook.

Note 2: We will have computerized homework assignments most days. **None accepted late!**

The daily class plan is to:
daily warmups
brief lecture, (ocassional long, boring tirades will hopefully be kept to a minimum).
maybe more drills
then go to the computer lab and work on the homework. (Almost never happens.)

1. Go over Course Syllabus carefully
2. Go over Rules for Exponents sheet
3. Do Worksheet # 1 Exponents I & II, 3 & 4
4. Go over problems pg 20 if time permits

Assign for today: P.2 hwk Rules for Exponents I & II

Up Next: Rational Exponents & Radicals
and Then: Linear Equations
A word about solving equations:

You don't always have to get the x on the left side!
Remember, tis easier to add than to subtract.

Solve for x:

$3 = 2 + x$

$10 - 2x = 0$

Assigned Bonus # 1 10 points: Age by Chocolate Problem.

MATH 112 Worksheet 1 Exponents I & II

Identify the base and the exponent.

1. $-x^5$

2. $(-x)^5$

3. 5^2

4. $(3xy)^2$

Simply each expression.

5. $a^n a^3$

6. $\frac{a^n}{a^4}$

7. $\frac{a^n a^3}{a^4}$

8. $a^n a^3 \;\; {}^{4}$

9. $2a^3 \;\; {}^{4}$

10. $\frac{b^n}{b^3} \;\; {}^{3}$

Simplify

1. $(\frac{a^2}{b^3})^4$

2. $\frac{a^{10}a^{-3}}{a^5 a^{-2}}$

3. $\frac{c^7}{c^2}$

4. $\frac{m^9 m^{-2}}{(m^2)^3}$

5. $(\frac{3x^5y^2}{6x^5y^{-2}})^{-4}$

6. $\frac{x^3y^{-4}w^5}{y^{-6}x^4w^5} \;\; {}^{2}$

7. $\frac{-3^{-2}a^3b^{-4}c^5}{6a^{-6}b^4c^5} \;\; {}^{-2}$

Your Age By Chocolate!

1. Pick the number of times a week you would like to have chocolate - more than one, but less than ten.

2. Multiply this number by two.

3. Add five.

4. Multiply by fifty.

5. If you have already had your birthday this year, add 1756. If you have not, add 1755.

Now subtract the four digit year in which you were born.

You should now have a three digit number. The first digit was your original number - how many times a week you would like to have chocolate. The next two digits are your age!

Note: 2006 is the only year for which this formula works, so you'll have to pretend it is this exact date three years ago.

Challenge: Figure out how the trick works and fix it so that it works for 2009 and write down the formula for it and turn it in next time at the beginning of class and

you'll get 10 points bonus!

This is **Bonus # 1** and is due August 19 at the beginning of class.

Math 112 Discussion 1b: P.3 Rational Exponents & Radicals

0. Pass out Sign - in Sheet.

1. look at the bonus# 1!

2. Drill with Worksheet #2 Exp # 3 & 4

3. Do problems from text pg 32 if time permits

    Refer to the Rules for Exponents page.

Assign for today: P.3 hwk and

Up next: P.4 & P5, polynomial arithmetic and factoring

Math 112 Worksheet 2 Exponents II

Instructions: If it's in exponential form, change it to radical form. If it is in radical form, change it to exponential form. Eliminate negative exponents.

1. $4^{\frac{1}{2}}$

2. $\sqrt[3]{a}$

3. $9^{\frac{1}{2}}$

4. $\sqrt{x^3}$

5. $4^{-\frac{1}{2}}$

6. $\sqrt[4]{x^3}$

7. $16^{\frac{3}{4}}$

8. $\sqrt[5]{\frac{x^3}{y^2}^{\,4}}$

9. $\frac{4}{9}^{\frac{1}{2}}$

10. $\sqrt{a^4b^6}$

11. $-\frac{8}{27}^{\frac{1}{3}}$

12. $\sqrt[4]{\frac{4^3}{9^2}}$

13. $-\frac{8}{27}^{\frac{2}{3}}$

14. .$\sqrt[4]{x^8y^{12}}$

15. $-\frac{8}{27}^{-\frac{2}{3}}$

Evaluate:

1. $4^{\frac{1}{2}}$

2. $\sqrt{4^3}$

3. $16^{\frac{3}{4}}$

4. $-\frac{8}{27}^{\frac{1}{3}}$

5. $4^{-\frac{1}{2}}$

6. $\sqrt[5]{\frac{x^{15}}{y^{10}}^{\,2}}$

7. $\sqrt[4]{x^8y^{12}}$

8. $4^{\frac{3}{2}}$

9. $-\frac{8}{27}^{-\frac{2}{3}}$

10. $\sqrt[4]{\frac{16x^4}{y^8}}$

Math 112 Discussion 2: P.4 Polynomials& P.5 Factoring

Make sure students can factor using grouping and the ac method.
Assign for today: Problems from P4 & P5, Quiz P1 - P6

Math 112 Discussion 3: P.7 Complex Numbers

0. Pass out Sign - in Sheet.

1. P.4 Complex Numbers - Brief Discussion

2. Do some problems on pg 71

Assign for today: P.7 hwk and internet Quiz P7: Complex Numbers

Up next: Section 1.1 Linear Equations

Math 112 Discussion 4: 1.1 Linear Equations I

0. Pass out Sign - in Sheet.

1. Questions on page 71?

2. Discuss idea of solving linear equations

3. Do 8,12,16,18,20,22,26,30,34,35,92,94
Them get up and do 11,19,25,29,39,83, 93

Assign for today: 1.1 hwk and internet Quiz 1.1 : Linear Equations

Up next: Section 1.1 Absolute Value Linear Equations

Solving Linear Equations

-What are they? Any equation that can be reduced to $ax + b = c$.

Solutions - What are they? Numbers that can be substituted into the original equation that yields a true statement.

The facts: Add, subtract, multiply and divide both sides of an equation by some number and check the solution.

The trick then, is to get an equation in a form in which one can quickly guess the correct solution.

Tirade on SOLVING LINEAR EQUATIONS

OBJECTIVE: Get x **alone** on one side of the = sign .

RULES: You can add, subtract, multiply or divide **both** sides of the equation by the **same** number without changing the answer.

TERMINOLOGY: You have two kinds of things to get rid of:

Terms- Things added to or subtracted from other things.
Factors- Things multiplying or dividing other things.

METHOD: **-** Always get rid of **terms** first, by adding their opposite to both sides.

- Get rid of factors next by performing the opposite operation on both sides.

- Each time you perform an operation on both sides, simplify each side before you proceed.

ADDITIONAL TECHNIQUES:

- When there are parentheseses on either side containing the objective variable, eliminate them first by using the distributive law.

- When there are fractions on either side, eliminate them before you do anything else by multiplying every term on both sides by the least common denominator.

- When x's occur in terms on both sides, eliminate them from one side first by adding the opposite of the **smallest** term containing x to both sides.

Always work in a vertical format. Each time you perform an operation, move down to the next line and write your new simplified version of the equation.

Examples:

1. $x + 3 = 5$

$$\begin{array}{rcl} x + 3 &=& 5 \\ \underline{\;\;3} & & \underline{-3} \\ x &=& 2 \end{array}$$

2. $x - 3 = 5$

$$\begin{array}{rcl} x - 3 &=& 5 \\ \underline{+3} & & \underline{+3} \\ x &=& 8 \end{array}$$

Or:

3. $x + 5 = 8$

$x + 5 - 5 = 8 - 5$

$x = 3$

4. $3x = 5$

$$\frac{3x}{3} = \frac{5}{3}$$

Note 1. : $3\cdot(1/3) = 1$. The factor here that needs eliminated is 3. So an alternate way to do this is to multiply both sides by 1/3.

5. Note 1 comes in handy when you have a problem like this one.

$$\frac{2x}{3} = 5$$

Since this is the same thing as (2/3)x, we can eliminate the factor 2/3 by multiplying both sides by its reciprocal, 3/2.

$$\frac{3}{2}\frac{2x}{3} = \frac{3}{2}\cdot 5$$

$$1\cdot x = \frac{15}{2}$$ or,

$$x = \frac{15}{2}$$

6. $3x + 4 = 6$

$$\begin{array}{rcl} 3x + 4 &=& 6 \\ \underline{\;\;-4} & & \underline{-4} \\ 3x &=& 2 \end{array}$$

$$\frac{3x}{3} = \frac{2}{3}$$

$$x = \frac{2}{3}$$

7. $2 - x = 7$
$\underline{-2 \quad\quad -2}$

$- x = 5$ Note 2: This little minus sign is **very** important. You must eliminate it. It is actually a factor of -1 !

$(-1)(-x) = (-1)(5)$ Get rid of it by multiplying both sides by -1.

$x = -5$

8. $2(3x - 4) + x = 5 - 2(5x - 3)$

$6x - 8 + x = 5 - 10x + 6$

$7x - 8 = 11 - 10x$ Now the **SMALLEST** term containing x is on the right side,
$\underline{+10x \quad\quad\quad +10x}$ so we'll eliminate it first.

$17x - 8 = 11$
$\underline{\quad +8 \quad +8}$
$17x = 19$

$$\frac{17x}{17} = \frac{19}{17}$$

$$x = \frac{19}{17}$$

9. **Never solve an equation containing fractions!** So, if the equation has fractions, step 1 is multiply every term in the equation by the common denominator and then reduce.

Finally, the bottom line is What is the solution ?

10. If you end up with an answer like 4 =4, or x = x, the solution is ALL REAL NUMBERS

11. If you end up with an answer like 4 = 6 or 0 = 5, the solution is NO SOLUTION.

12. Remember, if at anytime you multiply BOTH SIDES by anything containing the objective variable, you MUST check your answer in the ORIGINAL problem to make sure it "works", meaning, remember, $\frac{\text{anything}}{0}$ is undefined!

Example: $\frac{2x-3}{x-4} = \frac{5}{x-4}$, when you multiply both sides by x - 4 you get
2x - 3 = 5, which yields an answer of x = 4, BUT, upon substitution into the original problem yields $\frac{5}{0} = \frac{5}{0}$ WHICH IS UNDEFINED, so there is no solution to the problem

Math 112 Discussion 5: Absolute Value Equations

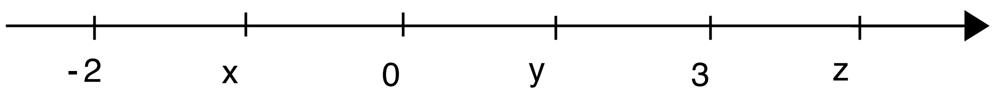
0. Pass out Sign - in Sheet.

1. Brief tiradeabout absolute value.

Absolute value - is a distance function. $|a - b|$ means the distance between a and b.
$|a| = |a - 0|$ = distance a is from 0.

Properties: $|ab| = |a|\,|b|$
$|a/b| = |a| / |b|$
$|a - b| = |b - a|$

Examples:



$|x|$, $|y|$, $|z|$, $|-y|$, $|-x|$, $|xy|$, $|-2x|$, $|y - x|$, $|3 - z|$, $|x - z|$, $|3 - y|$, $|3 - x|$

2. Absolute Value Equations: $| f(x) | = C$

Method I: solve $f(x) = C$ or $f(x) = -C$ (means you get two answers!)
$|x| = 5$

$|x - 3| = 5$

$|2x - 3| = 5$

Method 2: It's all about distance man.
demo this.
Note: Either way, you must get it into the form $|f(x)| = c$ first!

$2\,|x - 3| = 5$

$(2/3)\,|x - 3| = 5$

$6 - 4\,|x + 3| = -2$

Assign for today: 1.1 hwk

Up next: 1.2 Literal Equations

MATH 112 Worksheet 3 Absolute Value Equations

Solve each equation.

1. $|2x - 3| = 7$

2. $3 + |2x + 4| = 10$

3. $|x+6| = -6$

4. $5\,|4x + 3| + 7 = 2$

5. $8 - 5|5x + 1| = 12$

Math 112 Discussion 6: 1.2 Literal Equations

0. Pass out Sign - in Sheet.

1. Discuss literal equations

2. Do worksheet 4 on literal equations

3. Do more problems from text?

Assign for today: 1.2 hwk

Up next: 1.3 Points, distance & Circles

MATH 112 Worksheet 4 Literal Equations

1. $r_1 r_2 = r r_2 + r r_1$ for r

2. $r_1 r_2 = r r_2 + r r_1$ for r_2

3. $r_1 r_2 = r r_2 + r r_1$ for r_1

4. $d_1 d_2 = fd_2 + fd_1$ for d_1

5. $d_1 d_2 = fd_2 + fd_1$ for d_2

6. $b^2 x^2 + a^2 y^2 = a^2 b^2$ for a^2

7. $b^2 x^2 + a^2 y^2 = a^2 b^2$ for b^2

8. $S(1-r) = a - Ir$ for r

9. $Sn = \pi(n - 2)$ for n

10. $H(a + b) = 2ab$ for b

11. $y = \dfrac{2y - 3}{3x - 7}$ for x

12. $m_1 L_f + m_1 c_1(T-T_1) + m_2 c_2(T-T_2) = 0$ for T

Math 112 Discussion 7: 1.3 Pts, Distance & Circles

Worksheet on the Algebra & Coordinate Geometry of Circles

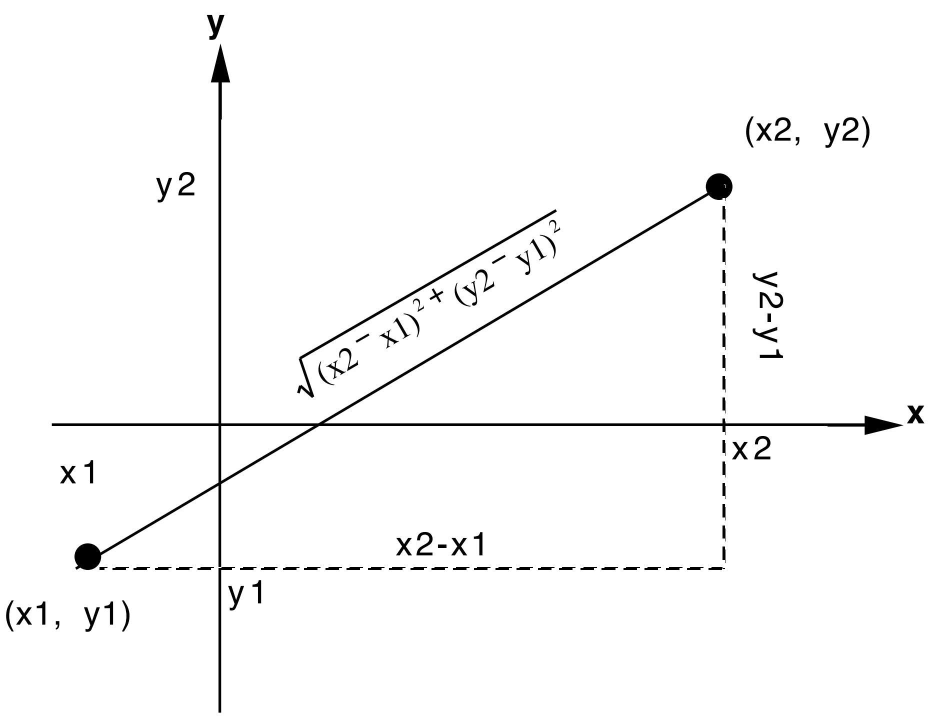
There are two objectives of this worksheet.

1. Know the forms and be able to recognize the equation of a circle.
2. Given enough information about a circle, be able to either
    a. Graph the circle or
    b. Write the equation of the circle

All of these objectives are obtained from two things, the distance formula and the definition of a circle.

The distance formula comes from
Pythagoras' theorem - The sum of the squares of the sides of a right triangle equals the square of the hypotenuse of said triangle.

From coordinate geometry, this becomes (see picture below)



So the distance between these two points becomes $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

The Definition of a Circle is - the set of all points in the xy plane a fixed distance r, (called the radius) from a fixed point(h,k) (called the center). Using the above definition of distance, then the point in the plane (x,y) is on the circle whose center is (h,k) and radius is r, if and only if
$\sqrt{(x-h)^2+(y-k)^2} = r^2$. Or, since both sides are always going to be positive, we could square both sides and have a less formidable looking representation

of a circle, $(x-h)^2+(y-k)^2 = r^2$

So any equation in the above form represents a circle with radius r and center (h,k)

Examples: Equation to Graph

$(x-3)^2 + (y-4)^2 = 25$ has center = (3,4) and radius r = 5 ;

notice the Domain = [3 -5, 3 +5]= [-2, 8) , and the range = [4 - 5, 4 + 5] = [-1, 9).

$(x+3)^2 + (y-1)^2 = 16$ has center = (-3,1) and radius r = 4

D = ?   R = ?

$x^2 + (y+4)^2 = 9$ has center = (0,-4) and radius r = 3

D = ?   R = ?

$(x-3)^2 + (y+5)^2 = 3$ has center = (3,-5) and radius r = √3

D = ?   R = ?

$(x-3)^2 + (y+5)^2 = -3$ does not represent a circle.

There are no ordered pairs of real numbers satisfying the equation. Any time you have an equation of the form $(x - h)^2 + (y - k) = C$ and the C is negative, the equation has no real solution.

Examples: Graph to equation

The circle with center (1, -4) and radius r = 4 has equation $(x-1)^2 + (y+4)^2 = 16$

The circle with center (-2, -5) and radius r = 1 has equation $(x+2)^2 + (y+5)^2 = 1$

The circle with center (0,0) and radius 8 has equation $x^2 + y^2 = 64$

The Big Picture of circles.

If any of the above equations were multiplied out and simplified, the result would look like

$Ax^2 + Ay^2 + Cx + Dy = E$, where C, D, and E are real constants. This is the **General Form** of a circle. So anytime you see an equation that can be expressed in this form, you must recognize it as the equation of a circle. You should also be able to write it in the **standard** form
$(x-h)^2 + (y-k)^2 = r^2$

from which you can find the center, radius, and graph, along with the domain and range. You will accomplish the conversion by

completing the square on both the x terms and the y terms.

Recall $(x + a)^2 = x^2 + 2ax + a^2$

so for a quadratic expression $x^2 + bx + c$ to be a binomial square, the constant term c must be $c = (b/2)^2$

To put it the other way around,

to make $x^2 + kx$ a binomial square,

we'd have to add the constant term $(k/2)^2$ to it.

Examples of General to standard.

Example 1: $x^2 + y^2 - 6x + 12y + 20 = 0$

First, rewrite the equation in the following form.

$x^2 - 6x +$ _________ $+ y^2 + 12y +$ __________ $= - 20 +$ ________ + ________

Next decide what goes in the blanks on the left by completing the square in both x and y expressions, and don't forget to include these terms on the right side as well to keep the equation valid.

Here's how to complete the square:

$(-6/2)^2 = (-3)^2 = 9$ and $(12/2)^2 = 6^2 = 36$, so the blanks above become

$x^2 - 6x + \mathbf{9} + y^2 + 12y + \mathbf{36} = - 20 + \mathbf{9 + 36}$, which, in standard form is written

$(x - 3)^2 + (y + 6)^2 = 25$

Whereupon we can now easily extract the center = (3, -6) and radius 5, which we could then graph.

D = ? R = ?

Example 2: $x^2 + y^2 + 8x - 6y + 9 = 0$

Solution:

$x^2 + 8x + \mathbf{16} + y^2 - 6y \mathbf{+ 9 =} -9 \mathbf{+ 16 + 9}$

$(x + 4)^2 + (y - 3)^2 = 1$, so

center = (-4, 3) and radius = 1

Example 3: $x^2 + y^2 + 8x - 6y + 8 = 0$

$x^2 + 8x + \mathbf{16} + y^2 - 6y \mathbf{+ 9 =} -8 \mathbf{+ 16 + 9}$

$(x + 4)^2 + (y - 3)^2 = 17$, so

center = $(-4, 3)$ and radius = $\sqrt{17}$

Example 4: $x^2 + y^2 + 8x - 6y + 25 = 0$

$x^2 + 8x + \mathbf{16} + y^2 - 6y \mathbf{+ 9 =} -25 \mathbf{+ 16 + 9}$

$(x + 4)^2 + (y - 3)^2 = 0$, so

center = $(-4, 3)$ and radius = 0, this circle is a single point, the center!

Example 5: $x^2 + y^2 + 8x - 6y + 29 = 0$

$x^2 + 8x + \mathbf{16} + y^2 - 6y \mathbf{+ 9 =} -29 \mathbf{+ 16 + 9}$

$(x + 4)^2 + (y - 3)^2 = -4$, so this circle doesn't exist, since $\sqrt{-4}$ is not a real number.

Problems:

1. $x^2 + y^2 - 2x - 10y = 55$ sol: $C = (1,5)$ and $r = 9$
2. $x^2 + y^2 + 4x + 10y + 13 = 0$ sol: $C = (-2, -5)$, $r = 4$
3. $x^2 + y^2 - 12x + 10y + 13 = 0$ sol: $C = (6, -5)$, $r = 4\sqrt{3}$

Math 112 More Discussion 7: 1.3 Pts, Distance & Circles

0. Pass out Sign - in Sheet.

1. Questions on page literal equations?

2. Pop quiz: Solve A(x -y) + 2 = Bx + C for x.

3. Point drill - Make sure they know proper way to draw a xy plane.

4. Distance: problems

5. Circles: Definition of a circle
Do worksheet 5

6. **Completing the square drill**

1. f(x) = x^2 + 2x + __ - ___ = (x + __ $)^2$ - ___

2. f(x) = x^2 + 6x + __ - ___ = (x + __ $)^2$ - ___

3. f(x) = x^2 + 8x + __ - ___ = (x + __ $)^2$ - ___

4. f(x) = x^2 + 10x + __ - ___ = (x + __ $)^2$ - ___

5. f(x) = x^2 + 12x + __ - ___ = (x + __ $)^2$ - ___

6. f(x) = x^2 + 16x + __ - ___ = (x + __ $)^2$ - ___

7. f(x) = x^2 + 20x + __ - ___ = (x + __ $)^2$ - ___

8. f(x) = x^2 + 14x + __ - ___ = (x + __ $)^2$ - ___

9. f(x) = x^2 + 2x + __ - ___ = (x + __ $)^2$ - ___

10. f(x) = x^2 + 4x + __ - ___ = (x + __ $)^2$ - ___

11. f(x) = x^2 - 6x + __ - ___ = (x + __ $)^2$ - ___

12. f(x) = x^2 - 8x + __ - ___ = (x + __ $)^2$ - ___

13. f(x) = x^2 - 10x + __ - ___ = (x + __ $)^2$ - ___

14. f(x) = x^2 - 12x + __ - ___ = (x + __ $)^2$ - ___

15. f(x) = x^2 - 16x + __ - ___ = (x + __ $)^2$ - ___

16. f(x) = x^2 - 14x + __ - __ = (x + __ $)^2$ - ___

17. f(x) = x^2 - 20x + __ - __ = (x + __ $)^2$ - ___

18. f(x) = x^2 + 3x + __ - ___ = (x + __ $)^2$ - ___

19. f(x) = x^2 - 5x + __ - ___ = (x + __ $)^2$ - ___

20. f(x) = x^2 + x + __ - ___ = (x + __ $)^2$ - ___

7. Do pg 121 44,46,48,50.

Assign for today: 1.3 hwk

Up next: Section 1.4 Equations of Lines

MATH 112 Worksheet 5 Distance & Circles

Find the distance between:

1. (1,3), (4, 5)
2. (-1, 2), (3, -4)
3. (3, -4), (3, 5)
4. (2,3) and(- 4,5)
5. (0,4) and 9,0)
6. (-2, 5) and (x, y)

7 (a, b) and (3, -6)

8 (h, k) and (x, y)

9. (a ,b) and (x, y)

10. (f, w) and (x, y)

Find equation of the circles

1. C = (3,4) r = 8
2. C = (- 2, 5), r = 2
3. C = (0, -3), r = 9
4. C = (4,0), r = 5
5. C = (5, -2), r = 1/2

Find center and radius of circles and sketch

1. $x^2 + y^2 - 2x - 10y = 55$ sol: C = (1,5) and r = 9
2. $x^2 + y^2 + 4x + 10y + 13 = 0$ sol: C = (-2, -5), r = 4
3. $x^2 + y^2 - 12x + 10y + 13 = 0$ sol: C = (6, -5), r = $4\sqrt{3}$

Math 112 Discussion 8: 1.4 Lines

0. Pass out Sign - in Sheet.

1. Questions on circles?

2. Do worksheet 6: slopes & lines

3. Can you find the slope of line between two points?

4. Can you graph a line using a point on it and it's slope? (Home-made)

5. Can you find it's equation using a point and it's slope?

6. Write in slope- intercept form and find slope & y-intercept (means solve for y)

7. Can you find it's equation using just two points?

8. Look at "special forms x = a and y = a. What is the slope of these two types?

9. Them do Warmup # 9: Slopes & Lines

8. if have time, do some

Assign for today: 1.4 hwk a

Up next: Test 1 Drill

MATH 112 Worksheet 6: Slopes & Lines

I. Find the slope between:

1. (1,3), (4, 5)
2. (-1, 2), (3, -4)
3. (3, -4), (3, 5)
4. (2,3) and(- 4,5)
5. (0,4) and 9,0)
6. (-2, 5) and (x, y)

7 (a, b) and (3, -6)

8 (h, k) and (x, y)

9. (a ,b) and (x, y)
10. (f, w) and (x, y)

II. Use the given slope and intercept to graph the line:

1. slope = 5, y-intercept = -3
2. slope = 1/5, y-intercept = 3
3. slope = -2/3, y-intercept = -1
4. slope = 0, y-intercept = -1
5. slope = undefined, x-intercept = -3
6. slope = 5/4, y-intercept = -2
7. slope = 0, y-intercept = 2
8. slope = 0, y-intercept = 8
9. slope = -5, y-intercept = 6
10. slope = -3/5, y-intercept = -3

III. Write the equation of the line with the given slope and intercept.

1. slope = 5, y-intercept = -3
2. slope = 1/5, y-intercept = 3
3. slope = -2/3, y-intercept = -1
4. slope = 0, y-intercept = -1
5. slope = undefined, x-intercept = -3
6. slope = 5/4, y-intercept = -2
7. slope = 0, y-intercept = 2
8. slope = 0, y-intercept = 8
9. slope = -5, y-intercept = 6
10. slope = -3/5, y-intercept = -3

IV. Write the equation of the line thru the given point with the given slope. (use point-slope form of the linear equation where possible.)

1. Point = (-1, 3) , slope = 8
2. Point = (-1, 3) , slope = 0
3. Point = (-1, 3) , slope = undefined
4. Point = (-1, 0) , slope = 2/3
5. Point = (1, 4) , slope = 3
6. Point = (-1, -3) , slope = -4
7. Point = (-2, 4) , slope =undefined
8. Point = (-2, 3) , slope = 0
9. Point = (4, -3) , slope = -3
10. Point = (0, 0) , slope = 5

V. Write in slope-intercept form and find slope and y-intercept. (means solve for y)

1. $2x + 3y = 0$
2. $2x + 3y - 4 = 0$
3. $2x - 4y - 6 = 0$
4. $y - 4 = 0$
5. $x = 5$
6. $x/3 + 5y = 3$
7. $2x - 3y = 2x + y - 5$
8. $5x + 2y/3 - 1/4 = 0$
9. $4x - 3 = 0$
10. $y - x = 0$

VI. Graph the line using the given point and slope.

1. Point = (-1, 3) , slope = 8
2. Point = (-1, 3) , slope = 0
3. Point = (-1, 3) , slope = undefined
4. Point = (-1, 0) , slope = 2/3
5. Point = (1, 4) , slope = 3
6. Point = (-1, -3) , slope = -4
7. Point = (-2, 4) , slope =undefined
8. Point = (-2, 3) , slope = 0
9. Point = (4, -3) , slope = -3
10. Point = (0, 0) , slope = 5

VII. Find the equation of the line thru the given points.

1. (1,3), (4, 5)

2. (-1, 2), (3, -4)

3. (3, -4), (3, 5)

4. (5, -2), (5, 5)

5. (-8, 8), (-8, -5)

6. (1, 2) , (3, 2)

7. (4, 9), (-3, 9)

8. (-2, 7), (7, 7)

9. (-1, -3), (5, 4)

10. (8, -2), (-4, -5)

Math 112 Test 1 Practice 6 pts each

Simplify and leave no negative exponents

1. $(-3/2)^{-}$

2. $x^{5}y^{-3}xy^{-2}x^{-3}y$

3. $(w^{-2}h^{3})^{-}$

4. $\left(\frac{-8x^4y^7}{16x^5y^6}\right)^3$

Evaluate:

5. $16^{\frac{3}{4}}$

6. $\sqrt[4]{\frac{16x^4}{y^8}}$

Write in a + bi form.

7. $\frac{3-2i}{1+i}$

Solve for x:

8. $2|3x-1|+4 = 8$

9. $3(x + 4) + 8 = 4 - 2(x + 4) - 5x$

10. Solve for r

S(1-r) = a - Ir for r

11. **Find the distance** between (-2, 3) and (-4 , 2)

12. **Find the equation of the circle** with

Center (2, -3) and radius 4 **and sketch it's graph**.

13. Find the center and radius of the circle whose

equation is $x^2 - 8x + y^2 + 12y = 12$

14. Find the slope between the points (-2,3) and

(-2,-5).

15. Graph the line y = (-3/5) x + 2 on an xy plane**.**

16. **Write the equation for the line that has:**

a. has no slope (undefined slope) and passes thru the point (2, -4).

b. slope 2/7 and passes thru the point (-2, -5)

c. has zero slope and passes thru the point (5, -3).

d. slope -2/3 and y intercept 4.

5 pts bonus solve for x

$$\frac{5}{x-3} = \frac{6}{x^2-9} - \frac{1}{x+3}$$

Math 112 Discussion 10: 1.6 Quadratic Equations

0. Pass out Sign - in Sheet.

1. Questions on Test 1

2. Solving Quadratic Equations: 3 methods.

Solve
Method 1: sq root method, don't forget the ± !
$x^2 = 9$

$(x - 3)^2 = 16$

$4(x+5)^2 = 16$

$3(2x - 7)^2 + 6 = 33$

Method 2: Factoring (after set in $ax^2 + bx + c = 0$ form)

a b = 0 means either a or b = 0

does ab = 12 mean either a or b = 12 ?

$x (x - 4) = 0$

$(x -2)(x +5) = 0$

$x^2 - 3x +2 = 0$

$4x^2 + 8x = 0$

Method 3: Quadratic Formula -when won't factor.

$x^2 + 2x + 4 = 0$

$3x^2 + 2x = 8$

$x^2 = 4x + 4$

General Steps to Solving Quadratic Equations

1. If have $ax^2 = b$, use sq root method.

2. Write in Standard Form $ax^2 + bx + c = 0$ (Why?)

3. If possible, factor and set each factor = 0 and solve for x.

4. If not factorable, use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Assign for today: Section 1.6 Hwk

Quadratic Equations Drill

Solve for x:

1. $x^2 - x - 6 = 0$

2. $3x^2 = -6x$

3. $4x^2 + x = 0$

4. $6x^2 = 3x$

5. $4x^2 - 12x = 0$

6. $x^2 = x$

7. $16x^2 + 9 = 24x$

8. $x^2 - 9 = 0$

9. $3x^2 + 13x = 10$

10. $x^2 + 9 = 0$

11. $x^2 + 16 = 0$

12. $x^2 - 36 = 0$

13. $9x^2 - 25 = 0$

14. $16x^2 + 27 = 0$

15. $(x-2)^2 = -3$

16. $(x+4)^2 - 1 = 0$

17. $x^2 - 4x + 1 = 0$

18. $x^2 - 4x + 7 = 0$

19. $9x^2 + 2 = 12x$

20. $9x^2 + 7 = 12x$

21. $9x^2 + 9x = 4$

22. $(3x + 2)^2 = -4$

23. $x^2 + 2x = 2$

24. $8x^2 + 3x = 0$

25. $\frac{2}{x} = \frac{3}{x^2} - 1$

26. $\frac{3}{x-1} - \frac{2}{x+3} = \frac{4}{x-2}$

Math 112 Discussion 11: Linear & Absolute Value Inequalities

0. Pass out Sign - in Sheet.

1. Questions on pg 157 ?

2. **Section 1.7**

linear inequalities:

Just like equations, only you must reverse inequality signs when divide or multiply both sides by a negative number.

Absolute Value inequalities - Two kinds

Yeah, Yeah, it's all about distance. But, how do we do the problems so we won't forget?

Simple, just solve it as an equation,
plot the points,
test points from each region in the original problem!

3. Do Worksheets # 7 & 8

4. linear inequalities

5. Unions & Intersections

Assignment for today: 1.7 hwk

Worksheet # 7 Unions & Intersections

† Union = "or" in english: combine all the elelments of both sets

‡ Intersection = "and" in english: choose only those elements that are in both sets.

Let A = {1, 2, 3, 4 }

Let B = {2, 4, 6,8}

Let U = Universe = {1,2,3,4,5,6,7,8,9,10}

A' = the complement of A = {5,6,7,8,9,10}

Write the indicated set.

1. A † B

2. A ‡ B

3. (B † A) ‡ A

4. (A † A') ‡ B

5. A' ‡ B'

6. B' † A'

7. B' and A

8. A ‡ (B'† A')

Express the answer both graphically and in interval notation.

1. $x > 3$ † $x > 5$

2. $x > 3$ ‡ $x > 5$

3. $x \leq -6$ ‡ $x \geq 2$

4. $x \geq -3$ † $x \leq -8$

5. $-3 < x < 8$

6. $2 > x \geq -1$

7. $\frac{2}{3} > x \geq -5$

8. $2 \leq x < 4$ ‡ $x \geq 4$

9. $2 \leq x \leq 4$ or $x \geq 4$

10. $1 < x < 3$ † $2 \leq x < 7$

11. $-4 < x \leq 5$ ‡ $0 \leq x < 8$

Write as a single interval if possible

1. [3 , 5) ‡ (1 , 7]

2. (-4 , 2] † (1 , 3]

3. (-∞, 4] ‡ (0, 8]

4. (-∞, -3] † (-10, ∞)

5. (∞ , 6] ‡ [3, ∞)

MATH 112 Worksheet # 8: Linear & Absolute Value Inequalities

Solve for x. Leave answer in **interval notation.**

Part I. Linear Inequalites

1. $3x + 2 < 5$
2. $5 - 2x \geq 3x + 4$
3. $2 - 3(x+2) \leq -4$
4. $8 < 3x + 2 < 10$
5. $-2 > 4 - 2x \geq -10$
6. $4 \leq \dfrac{2-4x}{5} < 8$

Part II. Absolute value equalities & inequalities

1. $|3x - 1| = 6$
2. $|2 - 5x| = 4$
3. $|x| < 5$
4. $|x| > 2$
5. $|x| > -2$
6. $|2x - 4| < -5$
7. $|3x - 4| \leq 2$
8. $|2 - 5x| < 6$
9. $|6x + 8| \geq 2$
10. $\left|\dfrac{3-2x}{5}\right| \leq 7$
11. $\left|\dfrac{5x+6}{2}\right| > 3$
12. $\left|\dfrac{1-x}{4}\right| \geq 0$

Functions Motivation

Do the pendulum trick (predict the period knowing only the length) use L = 8' and $T = 2\pi\sqrt{\frac{L}{g}}$, where g = 32 ft/s^2

In the 16^{th} century Galileo started an analytical thought process, others joined him. At first, ideas florished, but soon the complexities of the discussion became bottlenecked by the limitations of language. Precision and clarity has never been the strong suit of any human language. Out of necessity to clearly express their ideas, these scientists began to develop a new language that could not only express ideas and theories simply and concisely, but could be used to actually make new discoveries, without experimentation. Theories developed using this new language began to predict new discoveries before they happened experimentally! By the end of the 17^{th} century the language that evolved (and continues to evolve to this day), was so simple anyone could learn it, and so powerful that nearly every problem that could be stated in that language could be solved by it.

That language is of course Mathematics and is used today by all scientists world wide. Although not everyone actually uses this language in their day-to-day work, what has been discovered is that this learning to understand the principles of this language gives one a whole new method of thinking and reasoning called "Formal Reasoning" , which remains long after the principles of mathematics fade from memory.

The same process that enables one to predict the period of a pendulum when given only it's length has also produced the wonderful electronic age we all now enjoy and the space age we all anticipate. It has also created a world in which a knowlege of algebra is becoming increasingly necessary. Algebra is the subject of this course, so let's get on with it!

Math 112 Discussion 12: 2.1 & 2.2 Functions I

0. Pass out Sign - in Sheet.

1. Define a function

2.1 Tell whether or not have a function given, also find D & R:

- a. discrete set pg 183 43 - 44
- b. graphical pg 183 17-22
- c. algebraic pg 183 31-42
- d. Find D & R algebraically pg 183 45-54

2. Evaluate f(u) , Do:
Worksheet # 9
pg 184 probs 55-76

3. 2.2 Recognize "popular functions"

$y = ax + b$ linear
$y = ax^2 + bx + c$ Quadratic
$y = \sqrt{x}$ sq root
$y = x^3$
$y = \sqrt[3]{x}$
$x = y^2$
$y = |x|$

pg 207 probs 1 - 32

Assignment for today: 2.1 Hwk all, 2.2

Math 112 Worksheet # 9: Evaluating Function

Function evaluation: Find
f(2), f(-1), f(a), f(a^2), f(2b), f(2+b), f(x+h)

1. 1. $f(x) = x^2 - 4$

2. $f(x) = x^3 + 4x - 5$

3. $f(x) = x$

4. $f(x) = 2$

5. $f(x) = 3 - x^2$

6. $f(x) = 2 / x$

7. $f(x) = \sqrt{x}$

8. $f(x) = 4 / (x - 3)$

9. $f(x) = 5x / (3x + 4)$

10. $f(x) = 0$

11. $f(x) = \sqrt{5x}$

12. $f(x) = \sqrt{5 - x}$

Math 112 Discussion 13: 2.2 & 2.3 Piecewise & Transformations

0. Pass out Sign - in Sheet.

1. 2.2 Piece wise functions

   write piecewise fcts

   inc, dec, & more D & R

2. 2.3 Do worksheet 10

3. **Assignment today: 2.2 Hwk, 2.3 Hwk**

up next: 2.4 composite functions

Math 112 Worksheet # 10: Graphs, Domains

I. Graph and give D & R

1. $y = 3x + 2$

2. $y = x^2$

3. $x = y^2$

4. $y = x^3$

5. $x = y^3$

6. $y = \sqrt{x}$

7. $y = \sqrt[3]{x}$

8. $y = |x|$

9. $x = |y|$

3. $f(x) = x$

4. $f(x) = 2$

5. $f(x) = 3 - x^2$

6. $f(x) = 2 / x$

7. $f(x) = \sqrt{x}$

8. $f(x) = 4 / (x - 3)$

9. $f(x) = 5x / (3x + 4)$

10. $f(x) = 0$

11. $f(x) = \sqrt{5x}$

12. $f(x) = \sqrt{5 - x}$

13. $f(x) = \sqrt{5x - 2}$

13. $f(x) = \sqrt{x - 3}$

II. Find Domain

1. $f(x) = x^2 - 4$

2. $f(x) = x^3 + 4x - 5$

Math 112 Discussion 14: 2.4 Operations on Functions

0. Pass out Sign - in Sheet.

1. Do Problems

3. **Assignment today: 2.4 hwk**

up next: 2.5 Inverses of functions

Math 112 Discussion 15: 2.5 Inverses of Functions

0. Pass out Sign - in Sheet.

1. Any Questions about homework?

2. Define the inverse of a relation.

3. Definition: a function is 1 - 1 if its inverse is also a function

4. Demonstrate how to find the inverse of a relation it it is:

Descrete Home-made

Graphical Home-made

Algebraic

3. **Assignment today: 2.5 hwk**

up next: Test 2 Practice

Math 112   Test 2 Practice 5 pts each

Assignment today Internet Test 2

Solve for x.

1. $9 + 3\,|\,6 - 2x\,| = 18$

2. $5x^2 = 3$

3. $x^2 + 3x - 4 = 0$

4. $x^2 = 2x + 4$

Solve for x, graph the solution and also write it in interval notation.

5. $3x - 4 \leq 2 + 5x$

6. $4 < \frac{3 - 2x}{2} \# 6$

7. $2\,|\,x - 1\,| \leq 2$

8. $|\,2 + 9x\,| > 6$

9. If $f = \{(1,2), (-1, 2), (2,2)\}$
Domain =

Range =

Is the relation f a function?

10. Tell whether or not the equation represents a function.

a. $x^2 - y^3 = 8$

b. $x^3 + y^2 = 7y$

c. $x^2 = y^4$

d. $x + y^2 = 2x$

Find the domain for each function .

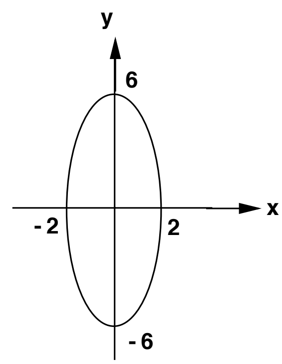
11. $f(x) = \frac{1}{x^2 - x - 6}$

12. $f(x) = \frac{1}{\sqrt{9 + 2x}}$

13. $\sqrt[5]{x^2 - 4x}$

14. $f(x) = -3x^2 - 4x^2 + 6x - 98$

15. Find the domain and range of the figure.



16. **Given $f(x) = 4 - x - x^2$, Evaluate and <u>simplify</u>**

$f(x - 3) =$

17. Describe how the graph of $y = x^2$ must be moved to obtain the graph of $y = (x-5)^2 + 3$

18. Given $f(x) = 5x - 7$, $g(x) = 4x^2 - 2x + 5$, Find:
<u>do not simplify!</u>

$g(f(x))$

19. Find the inverse $f^{-1}(x)$ for the function.

$f(x) = \frac{x + 1}{2x - 5}$

20. Solve the system of equations.

$2x - 3y = 4$
$x = 3y - 4$

Math 112 Discussion 16: 3.1 Quad fcts

0. Pass out sign in sheet.

1. Section3.1 quadratic functions (parabolas) 2 types you need to do.

Find vertex,
axis of symmetry,
max or min fct value
intervals on which inc & dec,
and sketch, by:

i) Factor & solve for roots to find x intercepts (if they are real) - home-made
ii) by completing the square and writing in $f = a(x-h)^2 + k$ form -,
discuss the $x = -b/2a$ thing for?

see drill sheet below!
2. given roots (x-intercepts) find equation. - home-made

3. Solving quadratic inequalities by:

i) graph & look - home-made
ii) making a sign chart (graph)

Today's Assignment: 3.1 Hwk

Worksheet # 11: Completing the Square on Quadratic Functions for the purpose of writing parabolic functions in $f(x) = a(x-h)^2 + k$ form so they can be graphed.

I. Completing the Square Drill

1. $f(x) = -x^2 + 2x$ + __ ___ = $(x$ + __ $)^2$ ___
2. $f(x) = -x^2 + 6x$ + __ ___ = $(x$ + __ $)^2$ ___
3. $f(x) = -x^2 + 8x$ + __ ___ = $(x$ + __ $)^2$ ___
4. $f(x) = -x^2 + 10x$ + __ ___ = $(x$ + __ $)^2$ ___
5. $f(x) = -x^2 + 12x$ + __ ___ = $(x$ + __ $)^2$ ___
6. $f(x) = -x^2 + 16x$ + __ ___ = $(x$ + __ $)^2$ ___
7. $f(x) = -x^2 + 20x$ + __ ___ = $(x$ + __ $)^2$ ___
8. $f(x) = -x^2 + 14x$ + __ ___ = $(x$ + __ $)^2$ ___
9. $f(x) = -x^2 + 2x$ + __ ___ = $(x$ + __ $)^2$ ___
10. $f(x) = -x^2 + 4x$ + __ ___ = $(x$ + __ $)^2$ ___
11. $f(x) = -2x^2 - 6x$ + __ ___ = $(x$ + __ $)^2$ ___
12. $f(x) = -4x^2 - 8x$ + __ ___ = $(x$ + __ $)^2$ ___
13. $f(x) = -5x^2 - 10x$ + __ __ = $(x$ + __ $)^2$ ___
14. $f(x) = 6x^2 - 12x$ + __ ___ = $(x$ + __ $)^2$ __
15. $f(x) = 8x^2 - 16x$ + __ ___ = $(x$ + __ $)^2$ ___

II. Graph parabolas I **Find vertex, axis of symmetry, max or min value.**

1. $f(x) = x^2$
2. $f(x) = 3x^2$
3. $f(x) = -x^2$
4. $f(x) = -2x^2$
5. $f(x) = x^2 + 3$
6. $f(x) = x^2 - 4$
7. $f(x) = -2x^2 + 5$
8. $f(x) = (x-2)^2$
9. $f(x) = (x+2)^2$
10. $f(x) = (x-5)^2$
11. $f(x) = 3(x-2)^2$
12. $f(x) = -2(x-2)^2$
13. $f(x) = 4(x-2)^2 + 3$
14. $f(x) = (x-2)^2 - 5$
15. $f(x) = -3(x-2)^2 + 6$
16. $f(x) = x^2 + 6x + 9$
17. $f(x) = x^2 - 6x + 9$
18. $f(x) = x^2 + 10x + 25$
19. $f(x) = x^2 - 12x + 36$
20. $f(x) = -x^2 + 6x - 9$
21. $f(x) = -x^2 - 6x - 9$
22. $f(x) = 2x^2 + 12x + 18$
23. $f(x) = 3x^2 + 30x + 75$
24. $f(x) = -5x^2 + 50x - 125$
25. $f(x) = x^2 + 6x + 10$

Math 112 Discussion 17: 3.2 Long & Synthetic div

0. Pass out Sign-in sheet.
1. Section3.2 Remainder theorem

sez P(c) = remainder upon division of P(x) by x-c
also, P(c) = 0 iff x - c is a factor of P(x)

2. Rational Root Thm

sez if $P(x) = a_n x^n + \ldots a_1 x + a_o$
then possible roots are of the form p/q, where p is a divisor of a_o and q is a divisor of a_n

3. Synthetic Division : show'em

Today's Assignment: 3.2 hwk

Math 112 Discussion 18: 3.3 & ~~3.4 Quad Types of Eq~~

0. Pass out Sign-in sheet.
1. Section 3.3 Given roots, find equation
2. ~~Section3.4 Miscellaneous Equations~~

~~4 or more terms~~
~~Misc~~
~~Quadratic Types~~
~~Re-drill on binomial square~~
~~Equations with Radicals~~

Today's Assignment: 3.3 & ~~3.4~~ hwk

Math 112 Discussion 19: 3.5 Graphing Poly eq's

0. Pass out Sign-in sheet.
1. Section 3.5 Graphing poly's in general.

i) Know how to find degree
ii) Know how to use this to determine shape of graph
iii) Know how to use exponents of factored poly to determine when graph crosses & when it bounces.

2. Worksheet #11 Graphing Polynomial functions

Today's Assignment: 3.5 hwk, internet quiz # 22

Math 112 Discussion 20: 3.6 Rational Fcts

0. Pass out Sign-in sheet.
1. Section 3.6 Rational functions

i) Know how to find x and y intercepts
ii) Know how to find vertical asymptotes (VA's)
iii) Know how to find horizontal asymptotes (HA's)
iv) Know how to sketch graph

Do worksheet # 12

Today's Assignment: 3.6 hwk

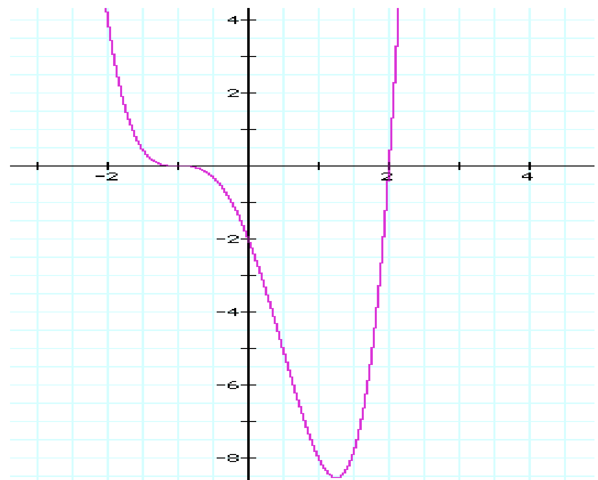
2. Get ready for test 3!

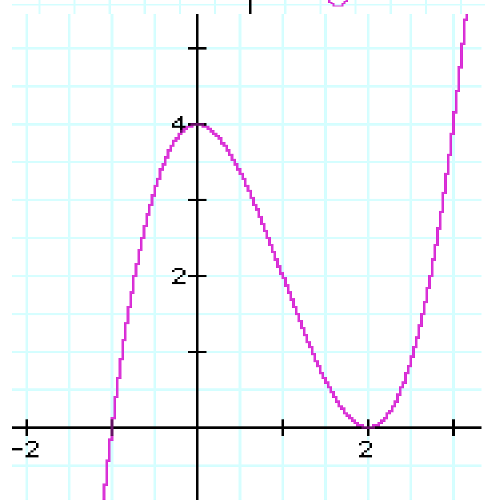
Math 112 Worksheet #11:   Graphing polynomials

Find x intercepts and sketch.

1. $y = x^2$
2. $y = x^3$
3. $y = x^4$
4. $y = x^5$
5. $y = x^6$
6. $y = x^7$
7. $y = x^8$
8. $y = (x-2)(x+4)(x+1)$
9. $y = (x-2)(x+4)(x+1)^2$
10. $y = (x-2)^2(x+4)(x+1)$
11. $y = (x-2)(x+4)^2 (x+1)$
12. $y = (x-2)^2(x+4)^2 (x+1)$
13. $y = (x-2)(x+4) (x+1)^2$
14. $y = (x-2)(x+4)^3 (x+1)^2$
15. $y = (x-2)(x+4)^3 (x+1)^2 (x-4)$
16. $y = (x-2)(x+4)(x+1)(x-4)(x-6)$
17. $y = (x-2)(x+4)(x+1)(x-4)(x-6)(x+7)$
18. $y = (x-2)(x+4)(x+1)(x-4)(x-6)(x+7)(x-8)$
19. $y = (x-2)^2(x+4)(x+1)(x-4)^2(x-6)(x+7)(x-8)$
20. $y = (x-2)^2(x+4)(x+1)^5(x-4)^2(x-6)(x+7)^3(x-8)$

Write equation for:





Math 112 Worksheet # 12: Worksheet on VA's & HA's
Graphing Rational Functions Using Vertical & Horizontal Asymptotes

There are basically two steps to this objective.
1. Finding HA's & VA's
2. Deciding where the graph goes .

Step 1, finding horizontal and vertical asymptotes, requires some understanding of a new number, infinity, which is denoted by the symbol ∞. Infinity is an unimaginably large number. You can think of it as the number at the end of the number line. When the value of a fraction becomes large without bound, we say it "**goes to infinity**". However, infinity is **NOT** a real number, so when a rational function "goes to infinity " at a particular value of x, we must say that it is not defined at that value of x, and **that value of x is not in the domain** of the function.

For rational functions, the value of the function goes to infinity for x values at which the denominator is zero and the numerator is not zero. If you were to draw a graph of the function near these x values, the graph would begin to look like a vertical line as x nears one of these values.

Illustration: look at the function $f(x) = \frac{1}{x-3}$

The denominator "has a zero" (which means the denominator equals zero) at x = 3. As x nears the value 3, from either side, the function values begin to get closer and closer to points on the line x = 3. So we say the line x = 3 is a **vertical asymptote** for this function.

In brief then, to find vertical asymptotes of a rational function, simply look for zeros of it's denominator.

Example 1: $f(x) = \frac{1}{x-4}$ has a VA at x = 4, and 4 is not in the domain of the function.

Example 2: $f(x) = \frac{x}{x^2-4}$ has two VA's, x = 2 and x = -2, and the domain of f is all reals except these two numbers.

Example 3: $f(x) = \frac{1}{x^2+4}$ has no VA's since $x^2 + 4 = 0$ has no real solution.

Problems: Find the VA's for

1. $h(x) = \frac{2x}{x-4}$, the answer is x = 4

2. $s(x) = \frac{2x^2 + 3x}{3x^2 - 48}$, the answer is x = 4 and x = -4

3. $p(x) = \frac{2x}{x^4 + 1}$. this function has no VA's

4. $f(x) = \frac{6x^4}{3x^2 - 2x - 5}$, the answer is x = 5/3 and x = -1

5. $r(x) = \dfrac{(2x - 1)(x+2)(x-6)}{x(x-3)(x+5)(x+8)}$, the answer is x =0, x = 3, x = -5, x = -8

Finding the horizontal asymptotes of a rational function.

Finding HA's requires asking the question, does the function value approach a real number as x goes to infinity? Finding the answer is easy if you remember a very important fact about polymomial functions, which is the dominance of the term of highest degree as x goes to infinity.

Illustration:

What this means is that as x gets large, $f(x) = 3x^4 - x^3 + 912x^2 + 4x - 87000$ begins to look like $g(x) = 3x^4$, simply because as x gets large, the value of $3x^4$ increases so much faster than the values of the other terms that their effect on the value of f (x) becomes insignificant.

So to find the horizontal asymptotes of a rational function, we simply throw away all but the leading term in numerator and denominator, reduce the resulting fraction, and evaluate it at x = ∞.

Example 1:

$g(x) = \dfrac{3}{x-4}$ becomes $g(x) = \dfrac{3}{x}$, which approaches zero as x goes to infinity, so the HA is y = 0

Example 2:

$g(x) = \dfrac{3x^2 + 42x - 9000}{x^2 - 4}$ becomes $g(x) = \dfrac{3x^2}{x^2}$ = 3 regardless of the value of x, so the HA is y = 3

Example 3: $k(x) = \dfrac{3x^3 + 42x^2 - x}{x^2 - 4}$ becomes $k(x) = \dfrac{3x^3}{x^2}$ = 3x , which approaches ∞ as x goes to infinity, so this function has no HA.

Problems: Find the HA for

1. $h(x) = \dfrac{2x}{x-4}$, the answer is h = 2

2. $s(x) = \dfrac{2x^2 + 3x}{3x^2 - 48}$, the answer is s = 2/3

3. $p(x) = \dfrac{2x}{x^4 + 1}$, the answer is p = 0

4. $f(x) = \dfrac{6x^4}{3x^2 - 2x - 5}$, the answer is no HA

5. $r(x) = \dfrac{(2x - 1)(x+2)(x-6)}{x(x-3)(x+5)(x+8)}$, the answer is r = 0

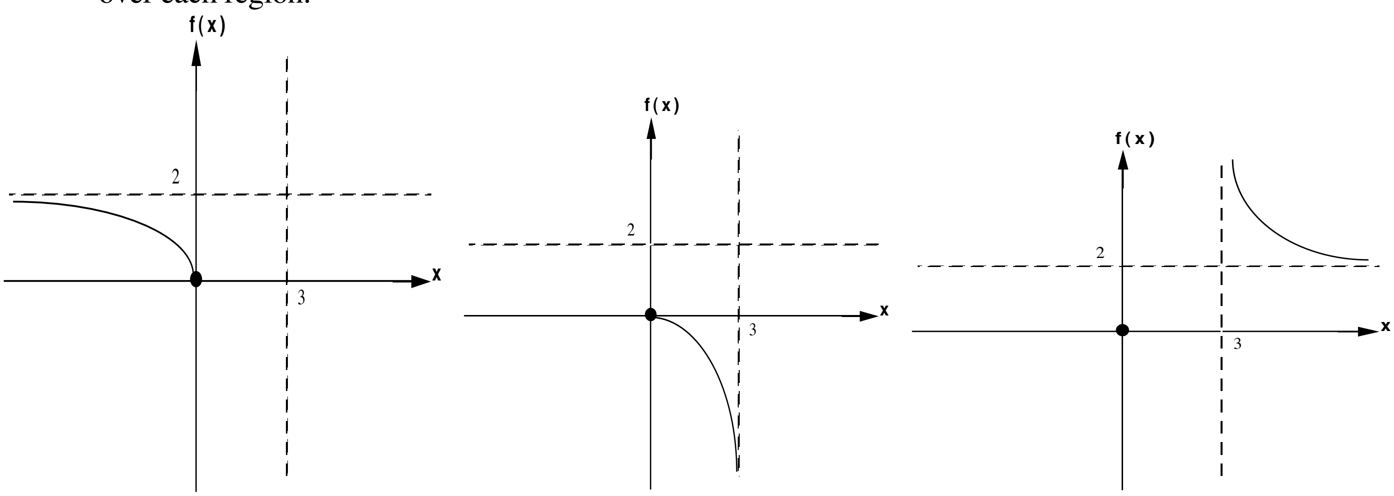
Step 2 : Finding where the graph goes, is the really big step. But, since rational functions are smooth continuous curves except where they are undefined, it is not as difficult as one would imagine. Usually, simply remembering this fact and using the x and y intercepts one can draw the curve without finding any other points. It is also helpful to remember that **the curve never intersects the VA's**, but **can intersect the HA's** as many times as is necessary. It is also highly recommended that you follow a set routine everytime you graph a rational function, and here is that routine.

Steps to graphing rational functions.

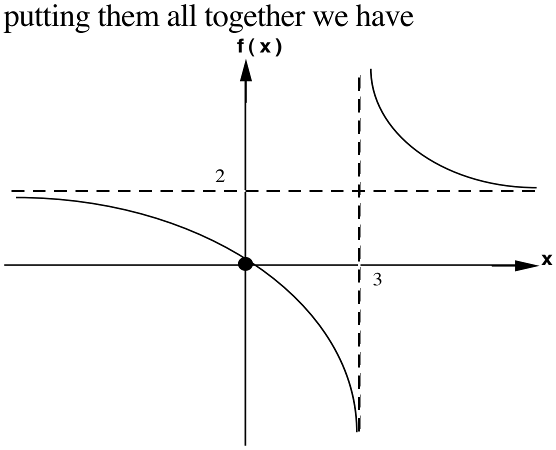
1. Find x intercepts (by replacing f(x) with a zero and solving for x.)
2. Find the function or y intercept (by replacing the x by zero and solving for f(x)).
3. Find VA's (by finding zeros of the denominator)
4. Find the HA (by eliminating all but the leading term in numerator and denominator and solving for f(x)).
5. Plot the intercepts, and draw in the VA's and the HA as dotted lines.
6. The x intercepts and VA's divide the x axis into regions. Beginning on the left end of the x axis, carefully draw the curve over each region, obtaining other sample points as necessary.

Example 1: $f(x) = \dfrac{2x}{x-3}$

- solving $0 = \dfrac{2x}{x-3}$ for x gives x intercept x = 0
- replacing x by zero in the function gives f intercept f(0) = 0
- Solving x-3=0 gives VA x = 3
- Reducing the function f(x) = 2x / x = 2 gives the HA f = 2
- The x intercept and VA divides the x axis into three regions, (-∞, 0), (0, 3), and (3, ∞). Here are the graphs over each region:

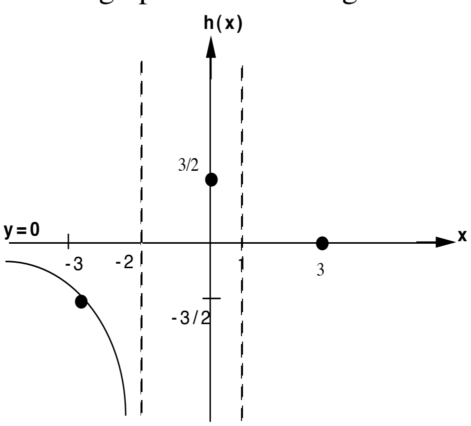


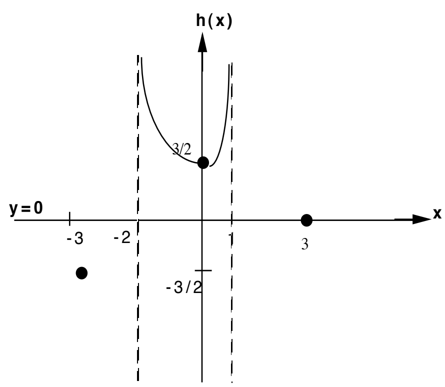
putting them all together we have

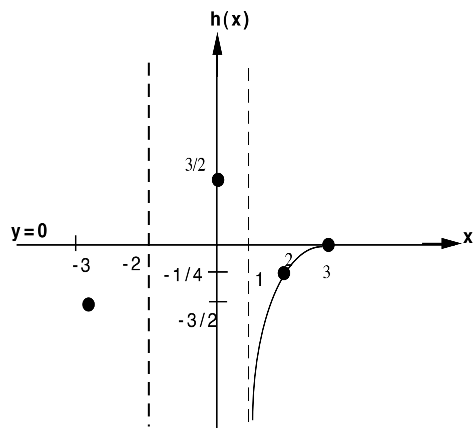


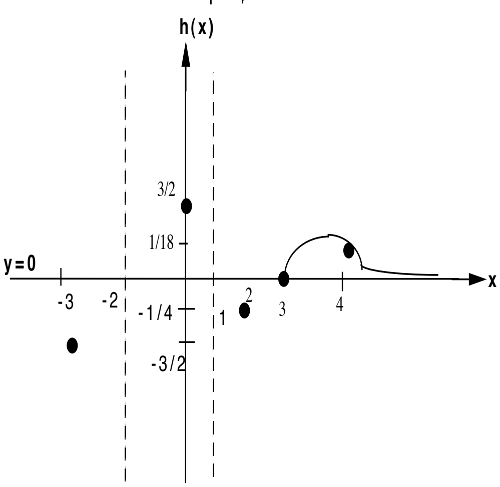
Example 2: $h(x) = \dfrac{x-3}{x^2+x-2}$

- Solving $0 = \dfrac{x-3}{x^2+x-2}$ for x yields x = 3 as the only x intercept.
- Finding h(0) by replacing x by zero yields the h intercept as h = 3/2
- Solving $x^2 + x - 2 = 0$ gives VA's x = -2 and x = 1
- Reducing the function $h(x) = x / x^2 = 1/x$ and evaluating as x goes to ∞, gives h = 0 as the HA
- The x intercept and VA's divide the x axis into 4 regions: (-∞, -2), (-2, 1), (1,3), and (3,∞). Here are graphs over each region.

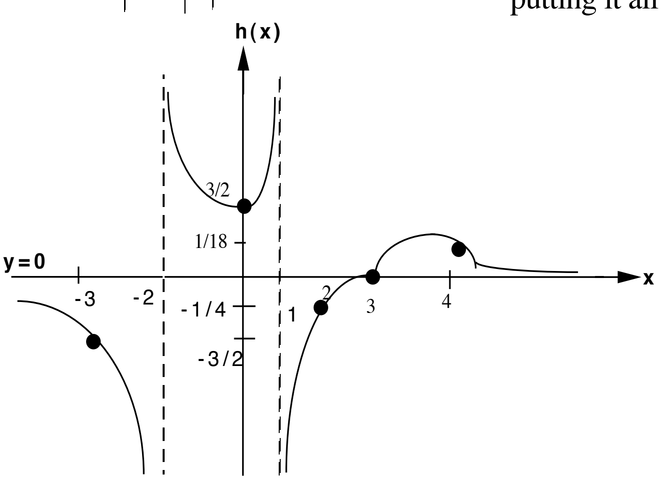








putting it all together we have:



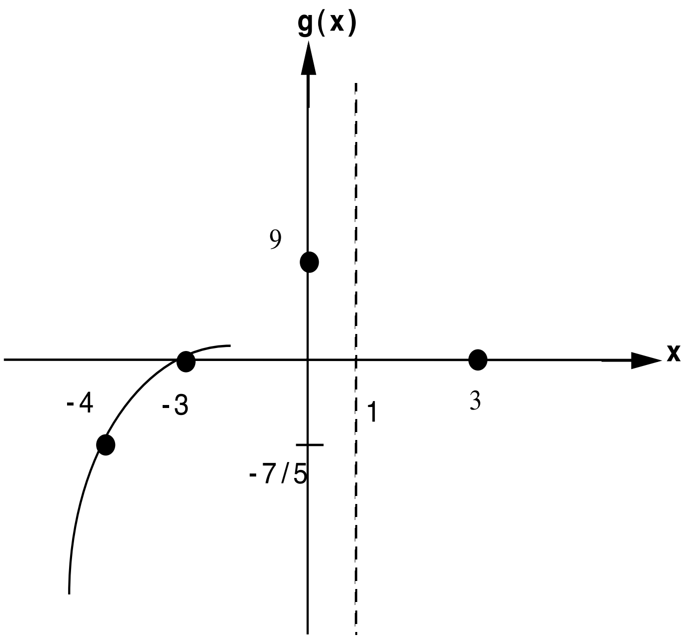
Notice that I had to find test points in the first, third and fourth regions, and note too that my graph is not to scale, I just made it so that the solution is clear.

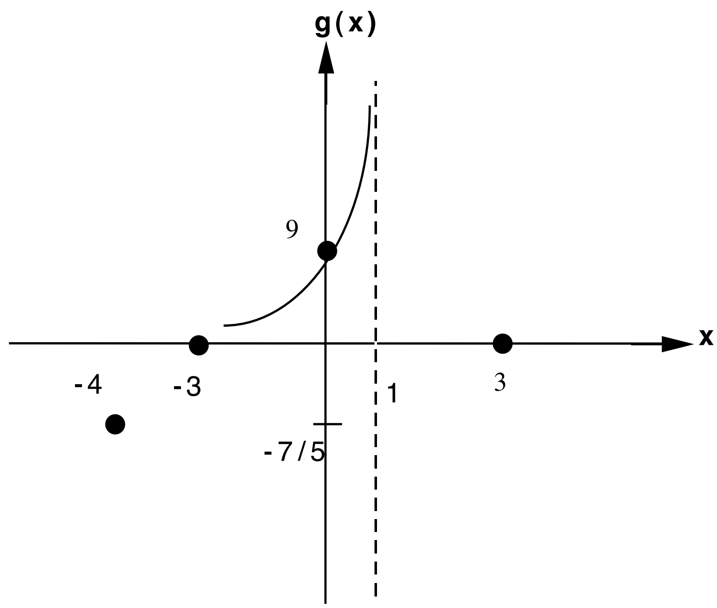
Example 3: $g(x) = \dfrac{x^2 - 9}{x - 1}$

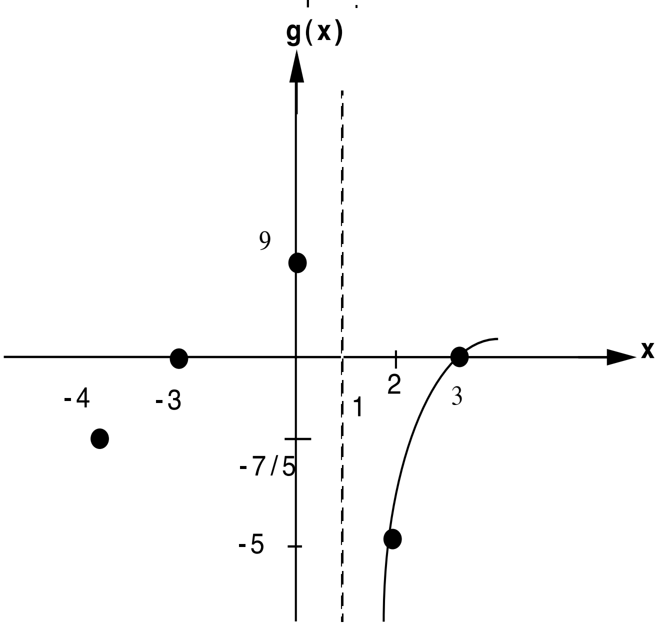
- Solving $0 = x^2 - 9$ yields $x = 3$ and $x = -3$ as x intercepts. (What happened to the denominator, why did I apparently throw it away?)

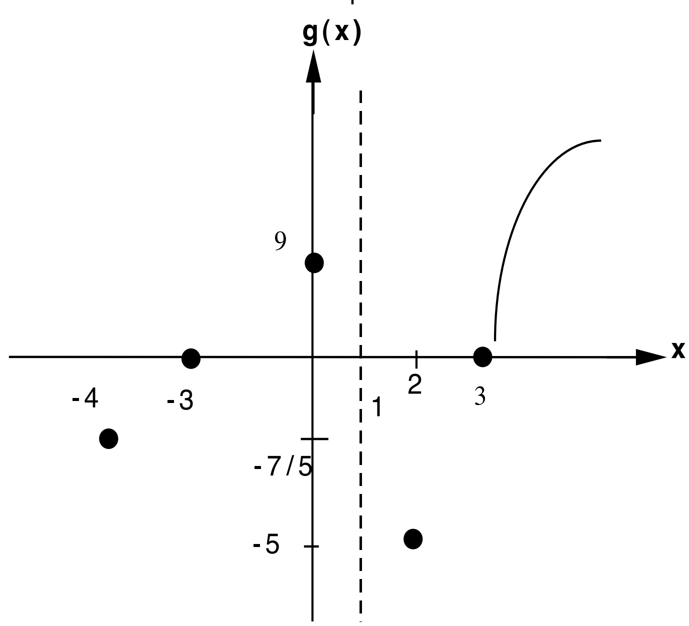
- Finding g(0) by replacing x by zero gives a g intercept of 9

- Solving $x - 1 = 0$ for x gives VA $x = 1$
- Reducing $g(x) = x^2 / x = x$ and evaluating as x goes to ∞ gives $g = \infty$, so there is no HA.
- The x intercepts and VA's divide the x axis into four regions , $(-\infty, -3)$, $(-3, 1)$, $(1,3)$, and $(3, \infty)$. Here are the graphs over each region.

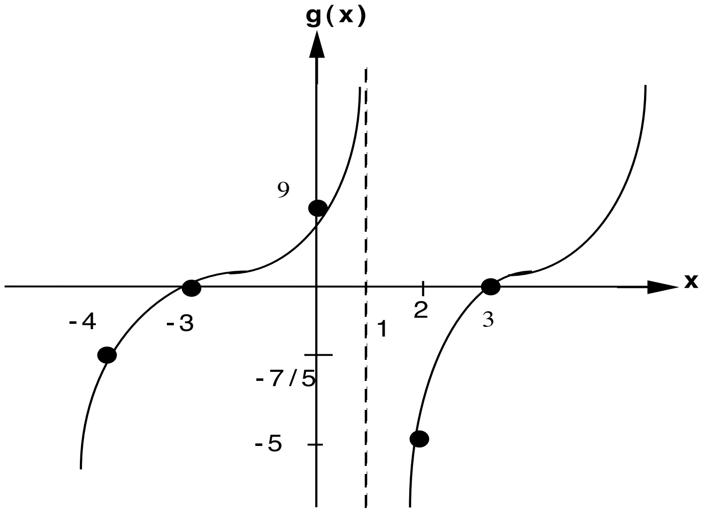








putting them all together looks like this



Problems: Find x & y intercepts, VA's & HA, and sketch

1. $f(x) = \dfrac{2x - 4}{x + 1}$

2. $h(x) = \dfrac{2x - 4}{x^2 - 1}$

3. $k(x) = \dfrac{2x^3 - 2x}{x - 2}$

Math 112 Test 3 Drill 6 Points each

(4 Points Absolutely Free!)

Write in $y = a(x-h)^2 + k$ form, then find vertex, axis of symmetry, max or min value.

1. $f(x) = 5(x - 4)^2 + 3$

2. $f(x) = 2x^2 + 12x + 9$

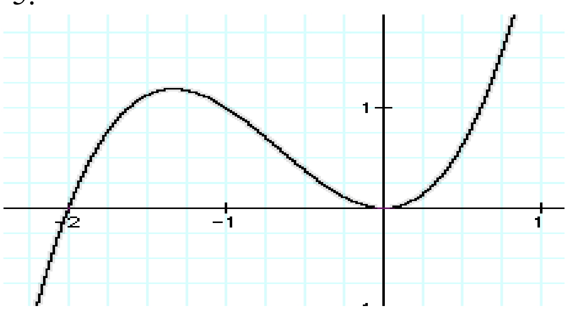
Graph:

3. $y \le -2(x+2)^2 + 4$

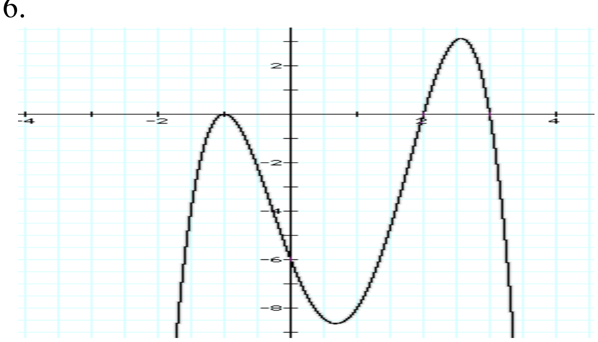
4. $y = (x - 2)^2 (x - 4)(x - 1)$

Find the equation for the function whose graph is:

5.



6.



7. Divide by long division

$$\frac{4x^4 + 6x^3 - 3x + 2}{x^2 - 3x - 1}$$

Use synthetic division to:

8. divide $3x^4 - 2x^2 + 5x - 1$ by $x + 4$

9. Evaluate $f(x) = 2x^4 - 2x^3 + 5x - 1$ at $x = 2$

10. Find the equation of the polynomial whose roots are: (you do NOT have to multiply out!)

a. -3, 2, 5, -1

b. 2, 3 - 4 i

Solve for x.

11. $x^3 - x^2 - 5x + 5 = 0$

12. $x^4 - 16 = 0$

13. $\sqrt{3x-2} - \sqrt{x-2} = 2$

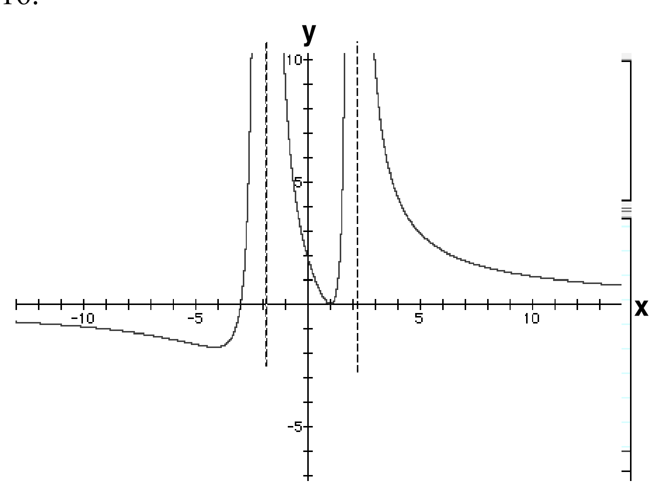
14. $\left(\frac{x+5}{2}\right)^2 - \frac{x+5}{2} - 6 = 0$

Find vertical asymptotes, horizontal asymptotes and sketch :

15. $y = \frac{3(x-2)^2}{(x+1)^2(x-3)}$

Write the equation for:

16.



Math 112 Discussion 21: 4.1 Exponential Fcts

0. Pass out sign in sheet.

I want you to learn three things today.

1. How to evaluate exponential expressions a^x.
2. How to graph basic exponential functions $y = a^x$.
3. How to graph $y = a\,B^{c\,x + d} + h$ and find it's domain and range.

Today's assignment: 4.1 Hwk

Math 112 Discussion 26: 4.2 Logarithmic Fcts

0. Pass out sign in sheet.

I want you to learn three things today.

1. How to solve exponential equations of type $a^{u(x)} = a^{v(x)}$

How does one solve $x + 3 = 5$

$3^x = 4$

2. How to evaluate logarithmic expressions .

3. How to convert log equations to exponential form and vice versa.

Today's assignment: 4.2 Hwk

Math 112 Discussion 23: 4.3 Laws of Logs & Log eqs

0. Pass out sign in sheet.

I want you to learn three things today.

1. Expanding log expressions

2. Contracting log expressions

3. Solve log equations by contracting and changing to exponential form.

Today's assignment: 4.3 Hwk

Math 112 Discussion 24: 4.4 Log & Exponential eqs

pg 403 6,8,15,19,20, 22

Today's assignment: 4.4 Hwk

Math 112 Discussion 25: Test 4 Review

Today's assignment: Internet Test 4

Math 112 Worksheet # 13 : Exponents

Evaluate the expression

1. 3^2
2. $4^{1/2}$
3. $8^{2/3}$
4. 3^{-2}
5. -2^{-2}
6. $(-2)^{-2}$
7. -4^2
8. $9^{3/2}$
9. $-9^{-3/2}$
10. -9^2
11. $(1/2)^2$
12. $(2/3)^2$
13. $(8/27)^{1/3}$
14. $(27/8)^{2/3}$
15. $(27/8)^{-2/3}$
16. $(16/81)^{1/4}$
17. $(16/81)^{-3/4}$

Graph and give domain and range

1. $y = 3^x$
2. $y = -3^x$
3. $y = 3^{-x}$
4. $y = (1/4)^x$
5. $y = 5^{-x}$
6. $y = -5^{-x}$
7. $y = (1/2)^{-x}$
8. $y = (2/3)^{-x}$
9. $y = 4^{-x}$
10. $y = -(4/3)^{-x}$

Graph & give domain and range

1. $y = 3^{2x}$
2. $y = -3^{3x}$
3. $y = 3^{-2x}$
4. $y = (1/4)^{5x}$
5. $y = 5^{-3x}$
6. $y = -5^{-2x}$
7. $y = (1/2)^{-8x}$
8. $y = (2/3)^{-x/2}$
9. $y = 4^{-x/3}$
10. $y = -(4/3)^{-x/4}$

Graph & give domain and range

1. $y = 3^{2+x}$
2. $y = -3^{3-x}$
3. $y = 3^{-x-2}$
4. $y = (1/4)^{5+x}$
5. $y = 5^{-x-2}$
6. $y = -5^{-2+x}$
7. $y = (1/2)^{8-x}$
8. $y = (2/3)^{-x-2}$
9. $y = 4^{x+3}$
10. $y = -(4/3)^{x-4}$

Graph and give domain and range

1. $y = 3^x + 2$
2. $y = -3^x - 4$
3. $y = 3^{-x} + 5$
4. $y = (1/4)^x - 3$
5. $y = 5^{-x} + 6$
6. $y = -(5^{-x}) - 7$
7. $y = (1/2)^{-x} + 3$
8. $y = (2/3)^{-x} - 9$
9. $y = 4^{-x} + 2$
10. $y = -((4/3)^{-x}) - 5$

Graph and give domain and range

1. $y = e^x$
2. $y = -e^x$
3. $y = e^{-x}$
4. $y = (1/e)^x$
5. $y = e^{-x}$
6. $y = -e^{-x}$
7. $y = (3e/4)^{-x}$
8. $y = (2e)^{-x}$
9. $y = (4e)^{-x}$
10. $y = -(e/3)^{-x}$

Interest Accrued (A) over (y) years on initial principle (P) at interest rate (r) if compounded (c) times each year

$$A_y = P(1 + r/c))^{yc}$$

$$\text{So } (1+ r/c)^{yc} = A_y/P$$

In order to solve this formula for y, we must:
First write the formula in log form.

$$\text{Log}_{(1+r/c)}(A_y/P) = yc$$

Next, using the change of base formula we have

$$\text{Log}_{(1+r/c)}(A_y/P) = \frac{\ln\left(\frac{A_y}{P}\right)}{\ln\left(1+\frac{r}{c}\right)} = yc\text{, from which we obtain}$$

$$y = \frac{\dfrac{\ln\left(\frac{A_y}{P}\right)}{\ln\left(1+\frac{r}{c}\right)}}{c}$$

If interest is compounded every instant of every day, (infinitely many times a year), this formula becomes

$$\underset{c\ 6\ 4}{\text{Limit}}\, P\left(1+\frac{r}{c}\right)^{yc} = Pe^{yr}$$

Math 112 19: Logs and Exponential Functions

1. Review of Logs. $\mathrm{Log}_a b = c$, what does it mean and why do we care?
One primary objective of math is to solve equations.
Examples: $3x + 5 = 4$
$x^2 - 2x - 15 = 0$
$\sin x = .5$
$3^x = 5$
The last two equations require a new method: Writing equations in inverse form.

i.e. to solve $\sin x = .5$, we write **"x is the angle whose sin is .5" as $x = \sin^{-1}.5$, or $x = \arcsin .5$** and to solve $3^x = 5$ we write **"x is the power to which 3 is raised to get 5" as $x = \log_3 5$.**

or, in general, $\log_{\text{base}} \text{answer} = \text{exponent}$

so, when you see a logarithmic function, think "exponent:.

The Natural log function and the number e.

$e = \lim\limits_{n\to\infty} (1 + 1/n)^n = 2.718281828...$ it is a non-repeating decimal, hence irrational.

$e^x = \lim\limits_{n\to\infty} (1 + x/n)^n$

$\log_e x$ is denoted ln x and is called the natural log function. $\log_e x \equiv \ln x$

Drill on saying and doing. **Solve for x:** $\log_{10}1000 = x$ $\log_8 2 = x$ $\log_4 0.5 = x$ $\log_5(1/125) = x$ $\log_2\sqrt{2} = x$ $\log_{10}1 = x$ $\log_b 1 = x$	$\log_x 16 = 4$ $\log_5 x = 3$ $\log_{.5} x = 2$ $\log_{125} x = 2/3$ $\log_x 27 = 3/2$ $\log_{10} x = 0$ $\log_5 0 = x$ $\log_3 (-9) = x$	**Laws of Logs:** 1. $\log ab = \log a + \log b$ 2. $\log a/b = \log a - \log b$ 3. $\log a^b = b \log a$ Change of base formula 4. $\log_a b = \dfrac{\log_c b}{\log_c a}$ **Solve for x:** 1. $\log x = 3\log 2 - \log 4$ 2. $\log_3 x = .5\log_3 5 + \log_3 4 - 2\log_3 3$ 3. $\log_2 x = \log_2 6 - .5\log_2 9$ 4. $\ln x = 3 \ln 2 - 2 \ln 3 - \ln 6$ 5. $\ln x^2 - \ln 2x = 3 \ln 3 - \ln 6$

MATH 112 WORK SHEET 20: Logs And Exponential Problems

Laws of Logs

1. $\log(ab) = \log a + \log b$
2. $\log(a/b) = \log a - \log b$
3. $\log a^n = n \log a$
4. $\log_a b = \dfrac{\log_c b}{\log_c a}$

Use Laws of Logs to Expand

1. $\log \sqrt[3]{x(1+x)}$
2. $\log x\sqrt{x-1}$
3. $\log \dfrac{x^3 y}{z}$
4. $\log \sqrt{\dfrac{x-1}{x+1}}$
5. $\log \dfrac{x-3}{x^2-4}^{3}$
6. $\ln \dfrac{x-3^{\frac{3}{5}} \quad x+2^{\frac{1}{2}}}{x+3^{\frac{1}{3}}}$
7. $\ln \dfrac{x^5 y^8}{\sqrt{z}}$

Contract using the Laws of logs

8. $\log 3 + \log 4 - 2\log 5$
9. $\log 4 + \log \pi + 3\log r - \text{Log}3$
10. $\log \pi + 2\log r$
11. $-2\log 5 + \log 13 + \log 2^3$
12. $.5[\log a - 3\log b + \log(c-a)]$
13. $-2\log 4 + 1/3\log 9 + 1/2\log 7$

Solve for x

14. $2^{x+1} = 128$
15. $30^{x+4} = 810000$
16. $(.01)^x = 3$
17. $4^{2x+1} = 7^{2x-1}$
18. $3^{2x+3} = 8^{3x-4}$
19. $\log(x+3)^2 = 2$
20. $\log(13+2x)^7 = 14$
21. $\log(3x+4) - \log(2x-2) = 2$
22. $\log(x^2-16) - \log(x-4) = 2$
23. $\log_2(x+1) + \log_2(x-1) = 3$
24. $\log x - \log(x+4) = .3010$
25. $x^{\log x} = 10$

Answer key to the logorithm worksheet

1. 1/3 log x + log (1 + x)

2. log x + 1/2 log (x-1)

3. 3log x + log y - log z

4. 1/2[log (x-1) - log(x+1)]

5. 3[log(x-3)-log(x-2)-log(x+2)]

6. 3/5ln(x-3) +1/2ln(x+2)-1/3ln(x+3)

7. 5ln x + 8ln y -1/2ln z

8. log(12/25)

9. $\log \frac{4\pi r^3}{3}$

10. $\log \pi r^2$

11. log(104/25)

12. $\log \sqrt{\frac{a(c-a)}{b^3}}$

13. $\log \frac{\sqrt[3]{9}\sqrt{7}}{16}$

14. 6

15. 0

16. -.2386

17. 2.978

18. 2.874

19. 7 or -13

20. 87/2

21. 204/197

22. 96

23. 3

24. no solution

25. 10 or .1

Math 112 Test 4 Drill 7 pts each

Today's assignment is Internet Test 4

Graph the function .

1. $f(x) = 3^x$

2. $f(x) = 3^{x-2} - 4$

3. $f(x) = -2^{-x}$

4. $f(x) = \log_{1/2} x$

5. $f(x) = \log_4 (x-3)$

6. $f(x) = \ln x$

Evaluate

7. $\log_x 16 = 4$

9. $\log_{125} x = 2/3$

10. $\log_x 27 = 3/2$

11. $\log_{10} x = 0$

12. $\log_5 0 = x$

13. $\log_3 (-9) = x$

Use Laws of Logs to Expand

14. $\log \sqrt[3]{x(1+x)}$

15. $\log x\sqrt{x-1}$

16. $\log \dfrac{x^3 y}{z}$

17. $\log \sqrt{\dfrac{x-1}{x+1}}$

<u>Contract using the Laws of logs</u>

18. $\log 3 + \log 4 - 2\log 5$

19. $-2 \log 5 + \log 13 + \log 2^3$

20. $.5[\log a - 3\log b + \log(c-a)]$

Solve for x:

21. $\log_3 x = .5\log_3 5 + \log_3 4 - 2\log_3 3$

22. $\ln x^2 - \ln 2x = 3 \ln 3 - \ln 6$

23. $\log (x + 3)^2 = 2$

24. $\log(3x + 4) - \log(2x - 2) = 2$

25. $30^{x + 4} = 810000$

26. $(.01)^x = 3$

27. $4^{2x + 1} = 7^{2x - 1}$

Attention Math 112 Class !

The Final is coming !
The Final is coming !

When? April 29, 2010 (8:00-10:00) section 1
May 3, 10:30 - 12:30 section 2

And what, you Ask, will be on the Final?????

Answer, 16 problems 12 points each 8 points absolutely free!

0. Simplify using rules for exponents similar to 1-6 on test 1
1. Solve a linear equation, similar to 8-9 on test 1.
2. Solve a linear inequality similar to one on test 2.
3. Solve a literal equation similar to one on test 1.
4. Find equation of circle similar to one on test 1

5. Solve a quadratic equation by factoring, similar to one on test 2.
6. Solve an absolute value inequality similar to ones on test 2.
7. Find the domain of a function whose equation is given similar to one on test 2.

8. Given two points: (Like one on test 1)
a. Find the distance between the points
b. Find the slope of the line between the two points.
c. Find the equation of the line passing through the two points.
d. Sketch the graph of a line.

9. Evaluate functions at various values similar to ones on test 2.
10. Sketch the graph of a parabolic function and find it's vertex, axis of symmetry, domain and range .

11. Find f^{-1} for a given f(x).
12. Solve quadratic type equation for x.
13. Sketch graph of given polynomial function or, given graph, find equation for that graph.
14. Use long division to divide polynomials similar to problem 7 on test 3

15. Use synthetic division to divide polynomials like #8 on test 3

16. Give roots, find equation of polynomial.

Show up and WIN 8 FREE points!

Math 112 Final Exam Drill 12 points each

Instructor: k.w.nicholson (Wow, 8 pts absolutely free!)

Solve for x:

1. $5x + (3 - 4x) + 2 = 5 - 2(3x + 2)$

2. $ax - b = c(x - 4)$

3. $2x^2 - x - 1 = 0$

4. $|2 - 9x| \leq 8$

5. $\left(\frac{x+5}{2}\right)^2 - \left(\frac{x+5}{2}\right) - 6 = 0$

6. **Given** points (-1,2) and (2,-3) **fnd:**

a. distance between the points

b. slope of the line between the points

c. equation of the line between the two points.

d. Sketch the graph of this line.

7. **Given** $f(x) = \frac{3-x}{2x+1}$**, find and simplify:**

a. f(1) =

b. f(-4) =

c. f(x+2) =

8. **Graph parabola and Find vertex, axis of symmetry, max or min value.**
$f(x) = 3x^2 + 12x + 80$

9. Solve the linear inequality. Put the answer in Interval notation.

$4 < 3 - 2x \leq 6$

10. Find the equation of the circle with center at (- 2, 4) and radius 6.

Find f^{-1} for this function.

11. $f(x) = \frac{2-x}{x+7}$

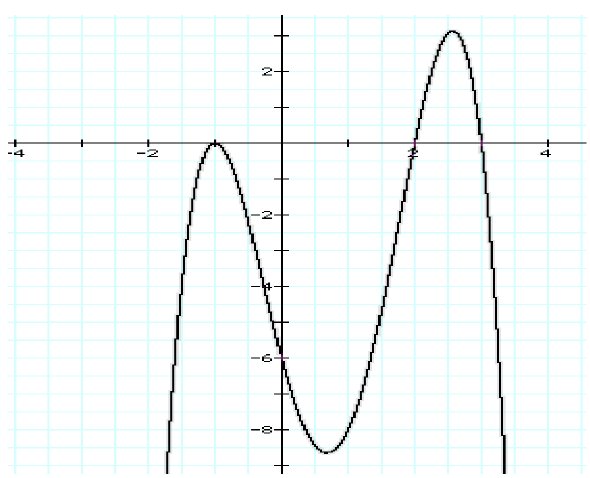
12. Sketch the graph of the function
$y = (x - 2)(x + 4)^2(x - 3)^2(x+1)^3$

13. Divide by long division
$$\frac{2x^4 + 6x^3 - 3x + 2}{x^2 - 4x + 1}$$
Use synthetic division to:

14. divide $3x^4 + 2x^2 - 5x - 1$ by $x + 2$

15. Find the domain of:
$f(x) = \frac{1}{\sqrt{5 - x}}$

16. Find the equation of the function whose graph is:



5 pts bouns!

Simplify and leave no negative exponents
-5^{-2}